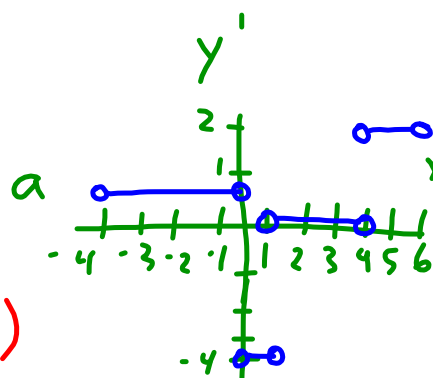
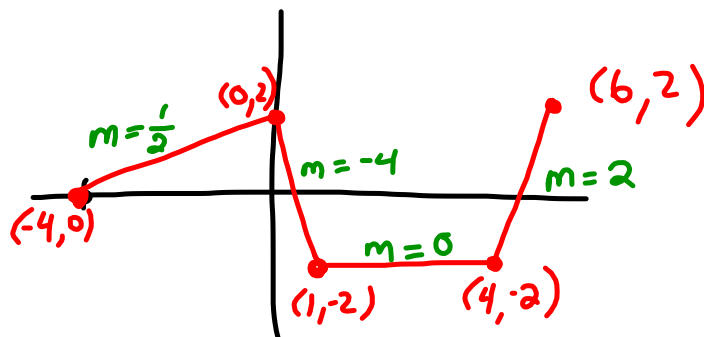


$$\textcircled{1} f(x) = \frac{1}{x}, a = 2$$

$\textcircled{26}$



b.

$$x = 0$$

$$x = 1$$

$$x = 4$$

Use the definition of a derivative to find the derivative of  $f(x) = 5x^2 - 8x$

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h \rightarrow 0} \frac{5(x+h)^2 - 8(x+h) - (5x^2 - 8x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{5x^2 + 10xh + 5h^2 - \cancel{8x} - 8h - 5x^2 + \cancel{8x}}{h} \\ &= \lim_{h \rightarrow 0} 10x + 5h - 8 = 10x - 8 \end{aligned}$$

Use the definition of a derivative to find the derivative of  $f(x) = \frac{-6}{x^2}$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \rightarrow 0} \frac{\frac{-6}{(x+h)^2} - \frac{-6}{x^2}}{h} = \lim_{h \rightarrow 0} \frac{\frac{-6}{(x+h)^2} \cdot \frac{x^2}{x^2} + \frac{6(x+h)^2}{x^2(x+h)^2}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-6x^2 + 6x^2 + 12xh + 6h^2}{x^2(x+h)^2} \cdot \frac{1}{h} = \lim_{h \rightarrow 0} \frac{12xh + 6h^2}{h \cdot x^2(x+h)^2}$$

$$= \lim_{h \rightarrow 0} \frac{12x + 6h}{x^2(x+h)^2} = \frac{12x}{x^2 \cdot x^2} = \frac{12x}{x^4} = \frac{12}{x^3}$$

## 3-2 Differentiability

### Learning Objectives:

I understand different ways that a function might be non-differentiable.

I understand how to find/graph derivatives on a graphing calculator at a given  $x$ .

I understand that differentiability implies local linearity and continuity.

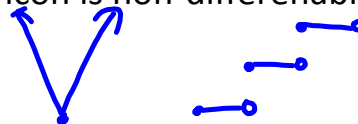
I can understand the Intermediate Value Theorem for derivatives.

### One Sided Derivaves

A funcon  $y = f(x)$  is differenable (the derivave exists) at a point  $x = c$  if and only if

$$f'(x) = \lim_{x \rightarrow c^+} \frac{f(x) - f(c)}{x - c} = \lim_{x \rightarrow c^-} \frac{f(x) - f(c)}{x - c}$$

In other words, the slope must be approaching the same thing on the le side as it is on the right side. If there is an abrupt change in the slope at some point  $x = c$ , that means that the funcon is non-differenable at that point.



**How could a funcon be Non-Differenable?**

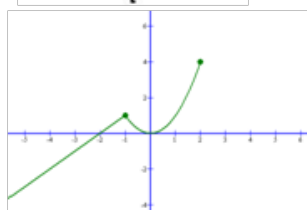
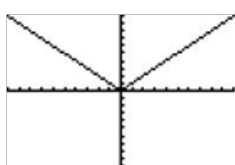
**What would it look like?**

## Corners and Cusps

```

Plot1 Plot2 Plot3
Y1=|X|
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=

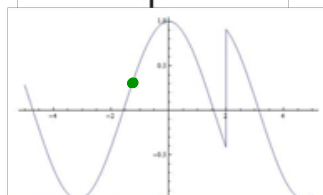
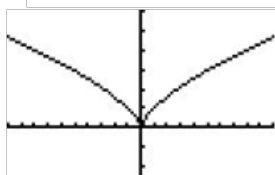
```



```

Plot1 Plot2 Plot3
Y1=X^2/3
Y2=X
Y3=
Y4=
Y5=
Y6=

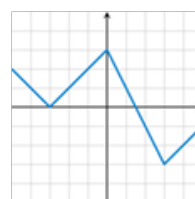
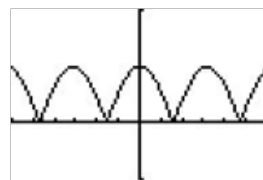
```



```

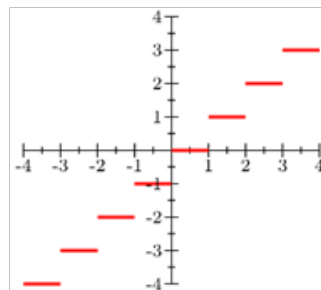
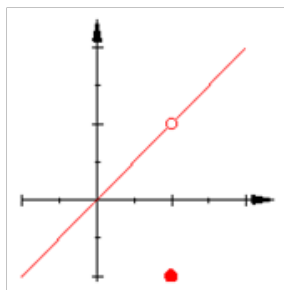
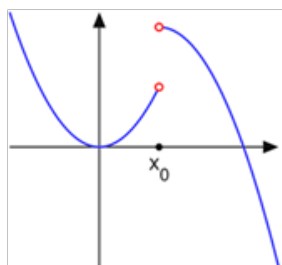
Plot1 Plot2 Plot3
Y1=|cos(X)|
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=

```



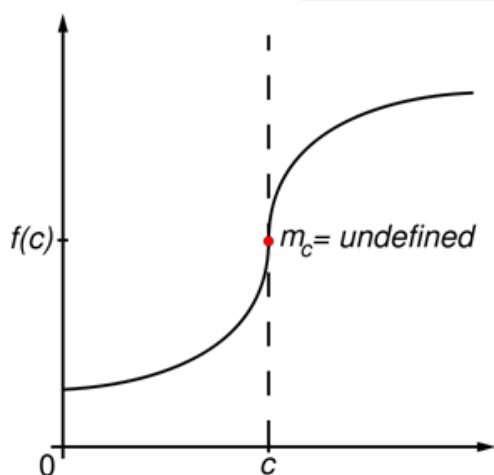
Remember, the derivave is really the slope. The slope is not approaching the same thing on both sides.

## Discontinuities



Remember, a derivative is really the slope of a tangent line. If there is no tangent line, there is no slope, there is no derivative.

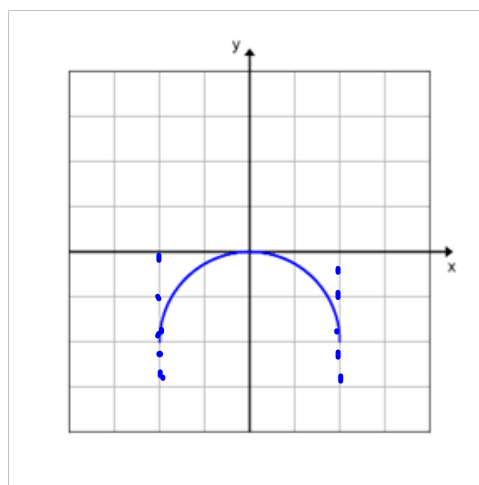
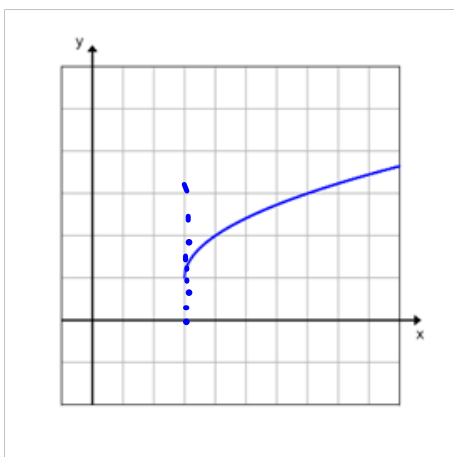
## Vertical Tangent Lines



Remember, a derivative is really a slope. A vertical tangent line has an undefined slope hence the derivative is undefined too. This case is different than the others in that there actually is a tangent line at the point in question – it's just that the slope of that tangent line isn't defined.



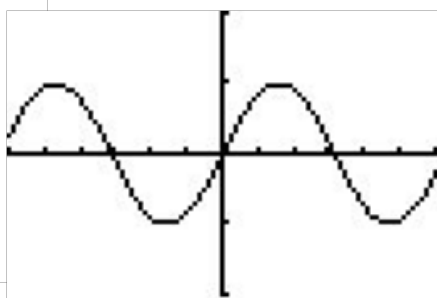
# Endpoints



Remember, a derivave is really the slope of a tangent line. If there is no tangent line, there is no slope, there is no derivave. The slope exists on one side but not the other.

# Differentiability Implies Local Linearity

```
Plot1 Plot2 Plot3
Y1 sin(X)
Y2 =
Y3 =
Y4 =
Y5 =
Y6 =
Y7 =
```

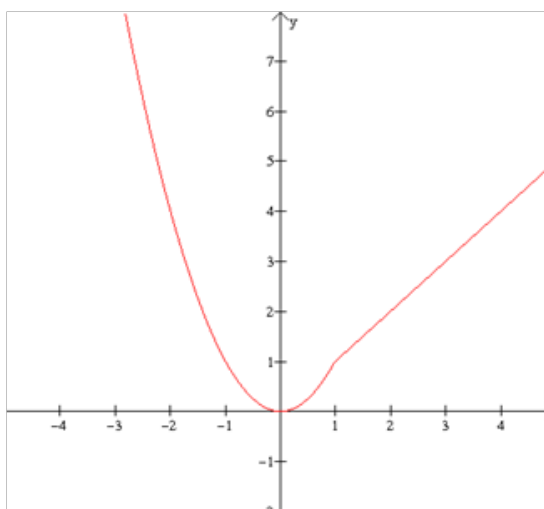


$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ x & \text{if } x > 1 \end{cases}$$

↓

$$y = x$$

$$m = 1$$



Ex1

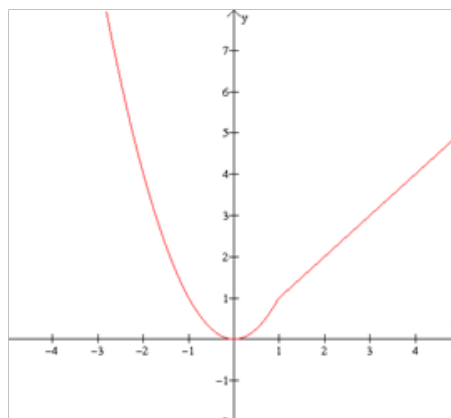
- a.) Do you think that this function is differentiable at  $x = 1$ ? Why or why not?
- b.) Find the right hand and left hand derivatives at  $x=1$ .

$$f'(x) = 2x \text{ at } x=1$$

$$m = 2$$

Since, the slopes are approaching different values on the left and right side of  $x = 1$ , the function is not differentiable at  $x = 1$ .

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ x & \text{if } x > 1 \end{cases}$$



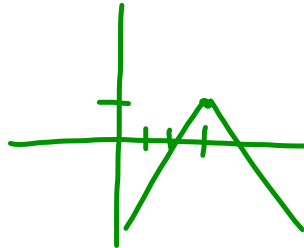
c.) Make it so that this function is differentiable at  $x = 1$ . You may only change 1 thing in the function.

Ex2. Find all points in the domain of  $f(x)$  for which  $f(x)$  is NOT differentiable. Identify why the function is not differentiable at each of these points.

1.)  $y = -2|x - 3| + 1$

$x = 3$

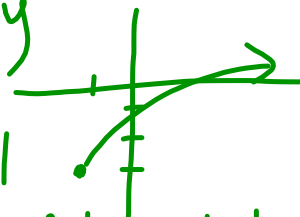
corner



2.)  $f(x) = \llbracket x \rrbracket$

not diff. at every integer

jump discontinuity



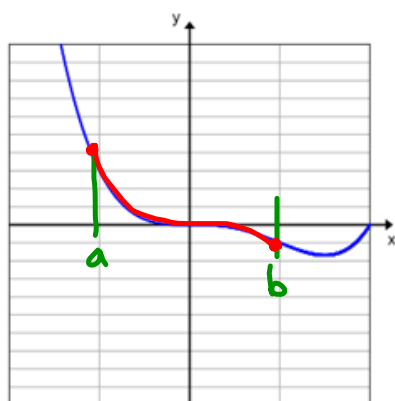
3.)  $g(x) = 2\sqrt{x+1} - 3$

$x = -1$

vertical tangent line

## Intermediate Value Theorem for Derivatives

If  $a$  and  $b$  are any two points in an interval on which  $f$  is differentiable, then  $f'$  takes on every value between  $f'(a)$  and  $f'(b)$  somewhere on the interval .



Ex3. Graph the derivave of each funcon  
the graphing calculator

1.  $y = x^2$

2.)  $y = \sin(x)$

# Homework

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